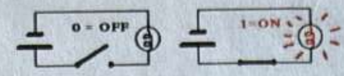
Kozlowski Konstantyn 353-mm

+0.3 to fin Grade

Massachusetts Institute of Technology (MIT)

Lecture by Pr. Bob Gallagher
Boole (1815-1864) & **Shannon** (1916-2001)

Sapere aude!

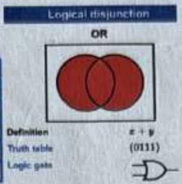


Logical addition (disjunction)

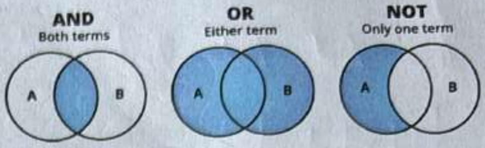
A	B	F=A∨B
0	0	0
0	1	1
1	0	1
1	1	1

Conjunction:

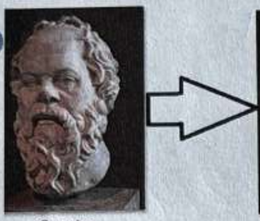
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F



BOOLEAN LOGIC



Good logic

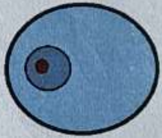


Socrates was a philosopher

$S \in \Phi$

philosophers are men

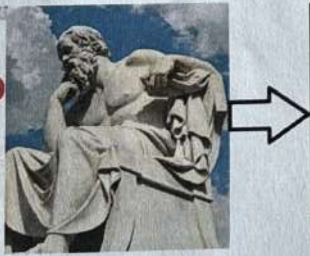
$\Phi \in A$



Socrates was a man

$S \in A$

Bad logic



Socrates was a man

$S \in A$

philosophers are men

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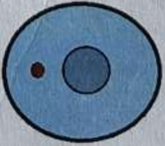
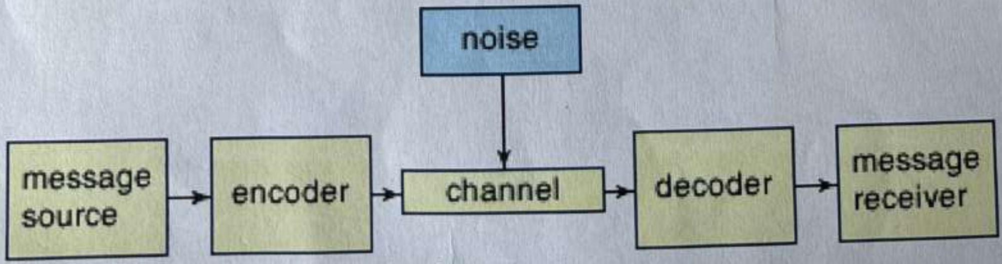


Socrates was a philosopher

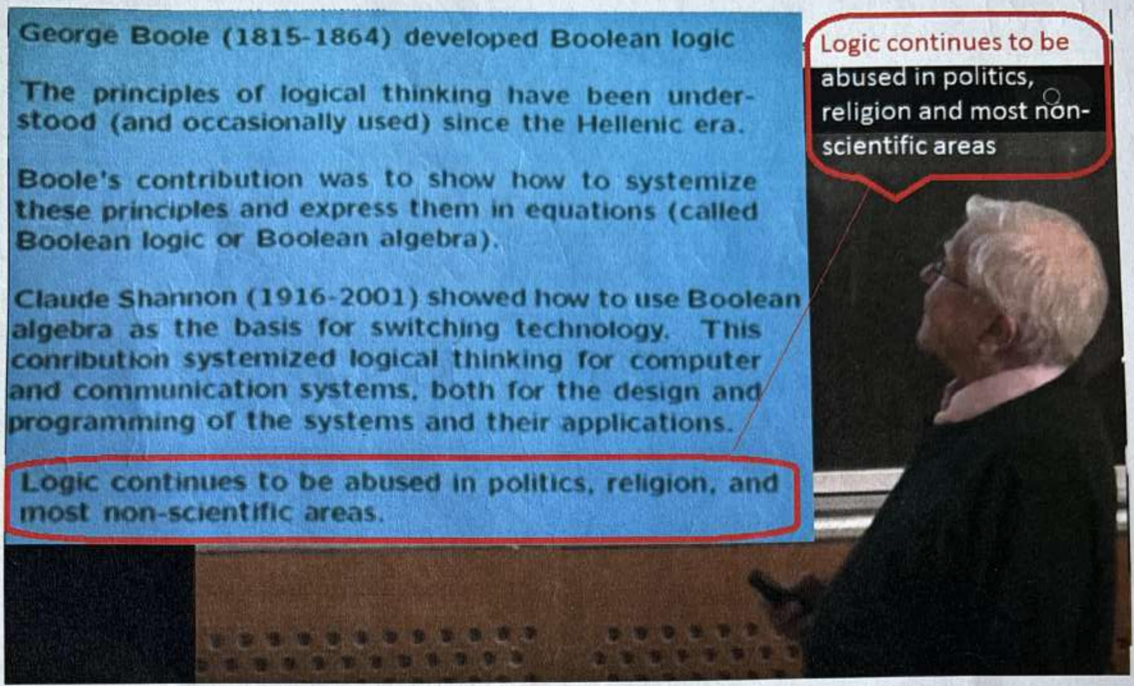
$S \in \Phi$



AO@dot.net.bg

Resume of Lecture by Pr. Bob Gallagher from MIT

Massachusetts Institute of Technology (MIT)



George Boole (1815-1864) developed Boolean logic

The principles of logical thinking have been understood (and occasionally used) since the Hellenic era.

Boole's contribution was to show how to systemize these principles and express them in equations (called Boolean logic or Boolean algebra).

Claude Shannon (1916-2001) showed how to use Boolean algebra as the basis for switching technology. This conribution systemized logical thinking for computer and communication systems, both for the design and programming of the systems and their applications.

Logic continues to be abused in politics, religion, and most non-scientific areas.

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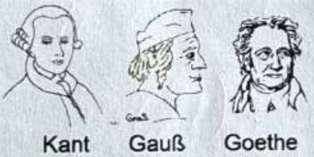
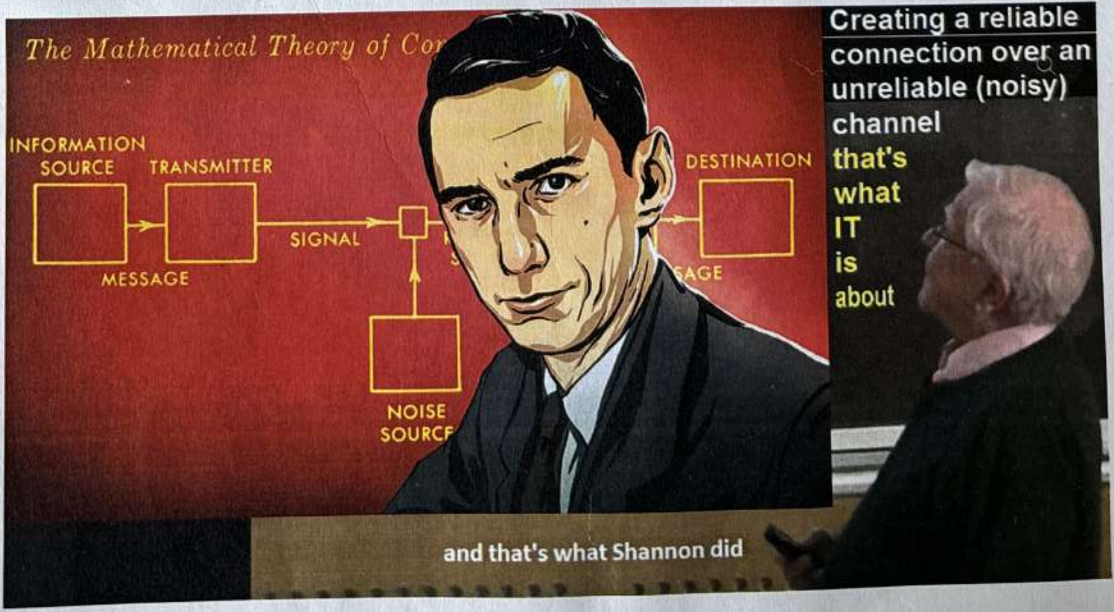


A little nationalistic, but this is an sample of right logic

Kant, Gauss, Goethe are great ⇒ Kant, Gauss, Goethe - Germans ⇒ Germany Great

Bad logic (abuse of logic)



Creating a reliable connection over an unreliable (noisy) channel that's what IT is about

and that's what Shannon did

1. How many ways to order the letters [illegible]

$n! = 6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6$

If we randomly reorder the letters what prob. that the vowels are all before the consonants

⊔ ⊔ ⊔ ⊔ ⊔ ⊔ ← probability

⊔ ⊔ ⊔
a n d ⎤
↑ ⎬ 2
or d n ⎦

a n d u
⊔ ⊔ ⊔ ⊔
a y n d
y a n d
a y d n
y a d n

$2 \cdot 2 = 4$

$n! = n \cdot (n-1)(n-2) \ldots 2 \cdot 1$ $m_1 + m_2 = n$

$$p(A) = \frac{m_1! \cdot m_2!}{n!}$$

How many arrangements

$A_1 A_2 A_3 BCD$

$BA_2 D A_1 A_3 C$

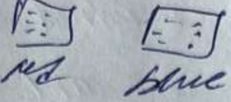
$BAD A_1 A C$

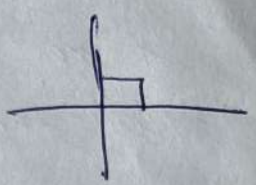
$BABAAC$

Events and probabilities

Ⓐ Tossing coin $\Omega = \{H, T\}$

Ⓑ Throwing a die





$\Omega = \{(i,j)\ \ 0,j \in [1,2,3,4,5,6]\}$

set of possible outcomes

A subset of Ω is called event

a) coin comes up tail $A = \{T\}$

b) $A = \{\underset{\text{red blue}}{(3;6)}\}$ - event

$w \in \Omega$ is the outcome, we say that A

occur if $w \in A$

Complement of A

A^c $\bar{A}$ occur when A doesn't occur

Set difference $A/B = A \cap B^c$

Intersection: $A \cap B$ occur if both A and B occur

$A \cup B$ occur if any A or B occur

Elementary combinations.

Arrangement of distinguishable objects

Suppose we have n distinguishable objects

$3! = 1 \cdot 2 \cdot 3$ $C_4^2 = \frac{4!}{2! \cdot 2!}$

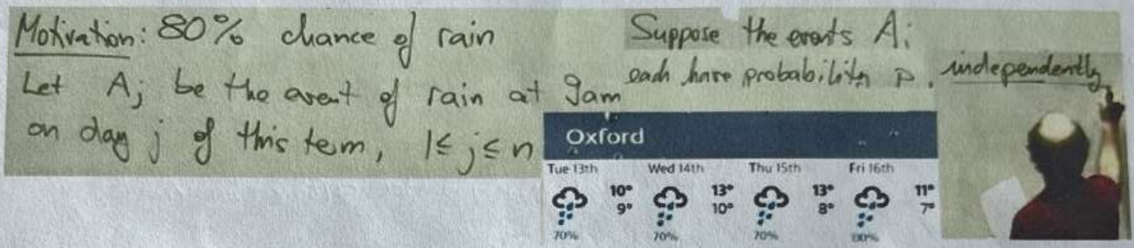
With prof. Matthias Winkel

Markoff Chain Probability Model for Oxford Weather

the next state only depends on the current state not the whole history

① listening
② first way of processing
③ **Writing**, incl. sth., you're not quite sure about

School ⬇ gravity ⬇ MOTION ==formalism==> University $E = MC^2$ $\oiint_S \ldots$ $\iint_S \vec{J}\,d\vec{S}$



Motivation: 80% chance of rain

Let A_j be the event of rain at 9am on day j of this term, $1 \le j \le n$

Suppose the events A_i each have probability p, independently

Oxford

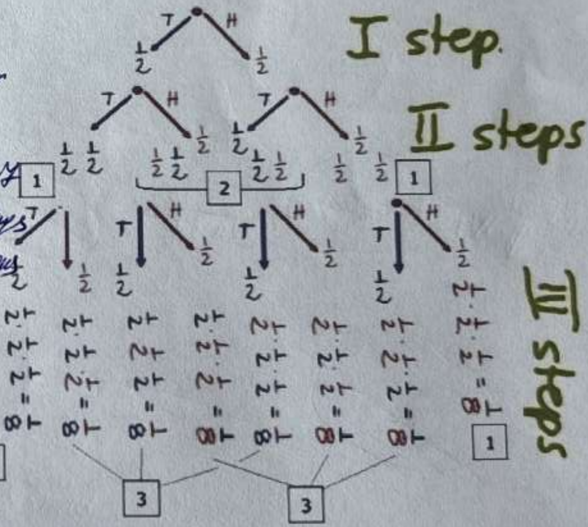
Tue 13th	Wed 14th	Thu 15th	Fri 16th
10° 9°	13° 10°	13° 8°	11° 7°
70%	70%	70%	80%

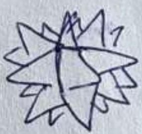
Pascal's triangle

Where each number is the sum of the two numbers directly above it

?: counting things, solving equations, figuring out probability

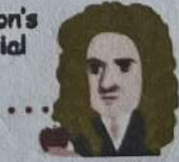
1 ← 1
1 1 ← 2
1 2 1 ← 4
1 3 3 1 ← 8
1 4 6 4 1 ← 16
1 5 10 10 5 1 ← 32
1 6 15 20 15 6 1 ← 64
1 7 21 35 35 21 7 1 ← 128

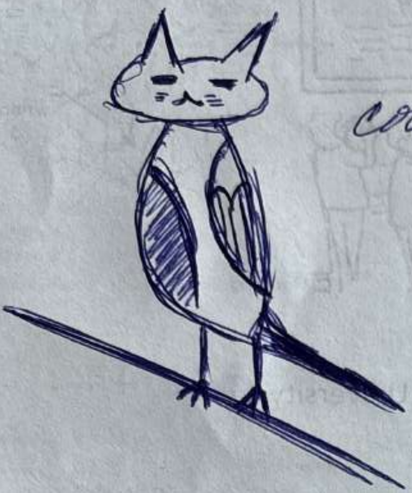




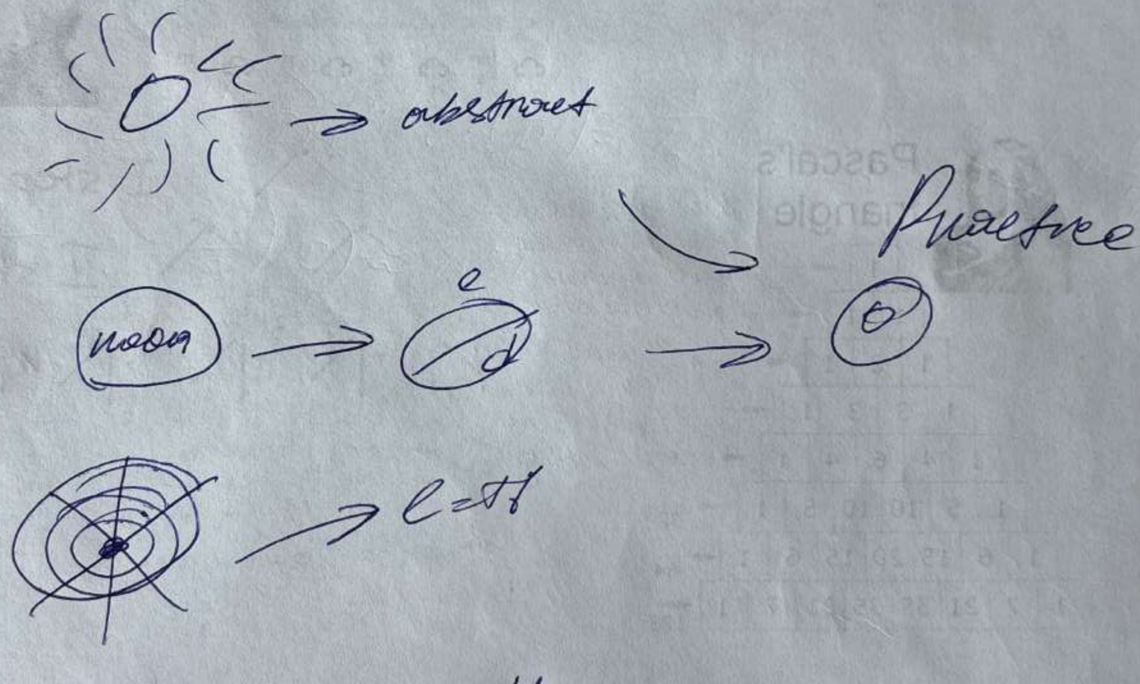
$$(a+b)^0 = 1$$
$$(a+b)^1 = a + b$$
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Newton's Binomial





catebird= catbird

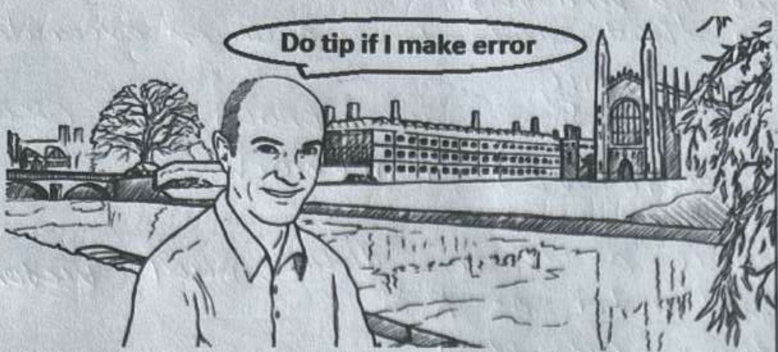


1 KR = 0,15 $

in 1998 = ...

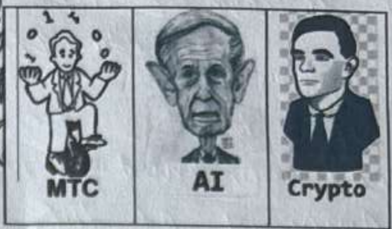
---

public void Bark() - prints "woof woof"
static void Main (string[] args]_ method where the prog. starts
public Dog() - we initialize objects with constructions
this.name.name - assign values
private, public, protected - All C# access levels
class AAtter - base class with protected and internal members
class Ason - inherits from AAtten

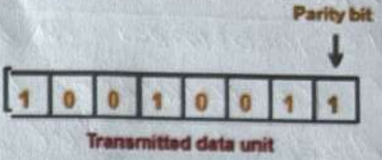


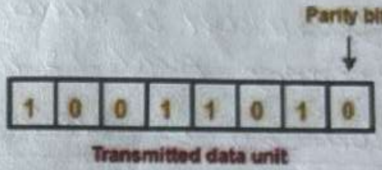
Sir Dr. D. MacKay,

University of Cambridge
(22 April 1967 – 14 April 2016)

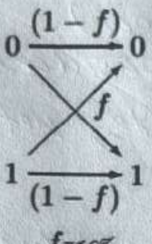


**"I believe in clean energy,
but I also believe in mathematics"**





visual metaphors
(error connection)



f=10%

adding extra inform. to a message...

adding redundancy in messages to ensure they can still be understood even with potential distortion or noise

R3R_BR3

triplication (a basic form of error-correcting code known as rep. code

repetition code

Source sequence s	Transmitted sequence t
0	000
1	111

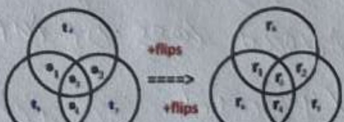
before transmission

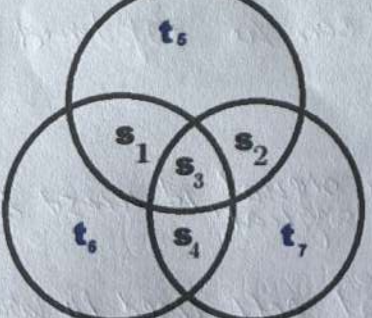
The repetition code R_3

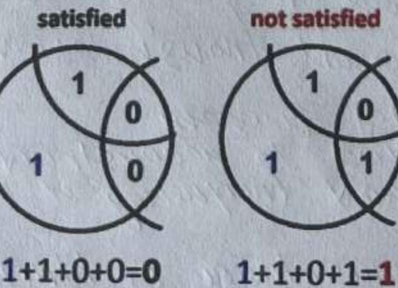
source message	s	0	0	1	0	1	1	0
	t	000	000	111	000	111	111	000
	n	000	001	000	000	101	000	000
	r	000	001	111	000	010	111	000
corrected errors			*					
undetected errors						*		

7.4. Hamming code.

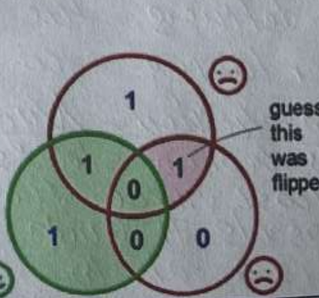
$$\frac{4}{\Sigma} \rightarrow \frac{7}{t}$$







1+1+0+0=0 1+1+0+1=1



7 - total number of bits after encoding
7 { 4 - number of data bits you want to send
  { 3 - remaining bits used for error checking

num 7 in bin: 0111
positions: 1234567
bits: p1p2d1p3d2d3d4
Final encoding
0001111

① csc.exe - com. line tool that compiles C#.code into a prog. that runs on Windows

② vbc.exe - compiles that code to encode an executable file or a library

③ MSCoreEE.dll - is a dynamic link library file associated with Microsoft. NET framework

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- **NR3** denotes the space of three-dimensional vectors with real coordinates represented as R3
- **Amount of Inf. theory (Hartley)**

Is measured in bans and is defined as the logarithm of the number of possible messages

- **Amount of Inf. theory (Shannon)**

Shannon's amount of inf. considers the probability of messages and is measured in bits, reflecting uncertainty

- **Concept of entropy**

Entropy is a measure of uncertainty of randomness of a random variable, indicating the average amount of information

- **Hartley definition of entropy**

The log of the num. of possible states of a system

- **Shannon entropy**

It generalizes Hartley's definition, taking into account the probabilities of diff. states.

- **Entropic compression of inf.**

This is the process of reducing the volume of data without loss of inf, based on statistical properties of the data.

- **Nyquist Interval**

It defines the max frequency that can be captured without overlaying the signal transmission

- **Kotelnikov - Nyquist theorem**

This theorem states that to reconstruct a signal, it must be sampled at least twice the max frequency of the signal.

- **Pascals Triangle**

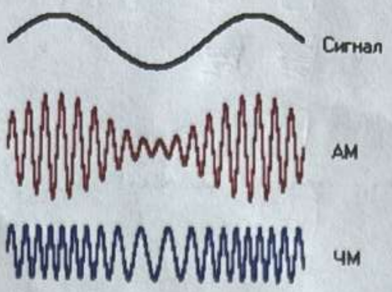
This is a tabular representation of binomial coeff. used in комбинаторика

- **Binomial distribution**

It describes the nums of success in a series of independent trials with two possible outcomes

Reginald A. Fessenden

(October 6, 1866 – July 22, 1932)

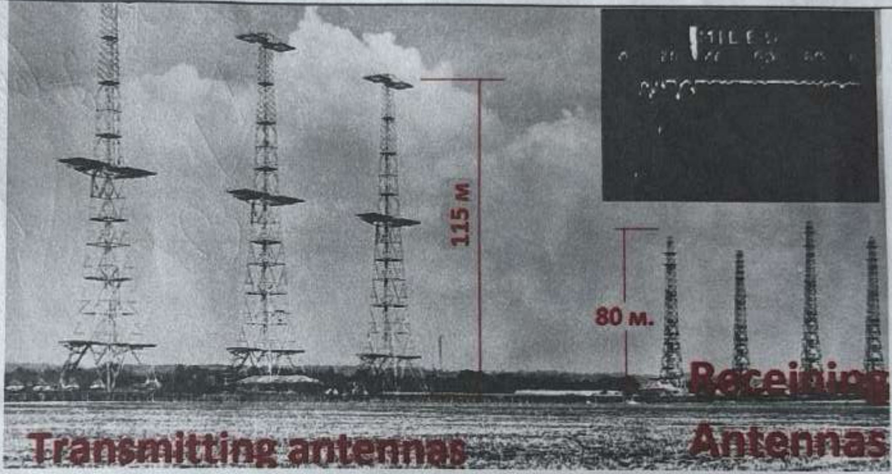


(October 6, 1866 – July 22, 1932)

first transmission of speech by radio (1900), and the first two-way radiotelegraphic communication across the Atlantic Ocean (1906)

"Ни одна организация, занимающаяся какой-либо конкретной областью деятельности, никогда не изобретает какие-либо важные разработки в этой области или не внедряет какие-либо важные разработки в этой области до тех пор, пока она не будет вынуждена сделать это из-за внешней конкуренции.." Oxford University Press. The Quarterly Journal of Economics , Feb., 1926, p. 262.

Battle of Britain
(3 month 3 weeks)
10.07-31.10.1940



Radar played a major role in the Battle of England

H. Nyquist

$$W = K \log m$$

Where W is the speed of transmission of intelligence,
m is the number of current values,
and, K is a constant.

Ralph **Hartley**
(81:1888-1970)

$$H = n \log s$$
$$= \log s^n.$$

0,5 - 1
0,25 - 2
0,125 - 3
0,125 - 3

0,5
0,5
0,125
0,75

$\boxed{1{,}75}$

$\frac{2}{8} \cdot \frac{1}{4} \to 25\%$

$\frac{1}{8} - 12{,}5\%$

$\frac{1}{8} - 12{,}5\%$

white ①

black ②

2 вопроса

☐ ③

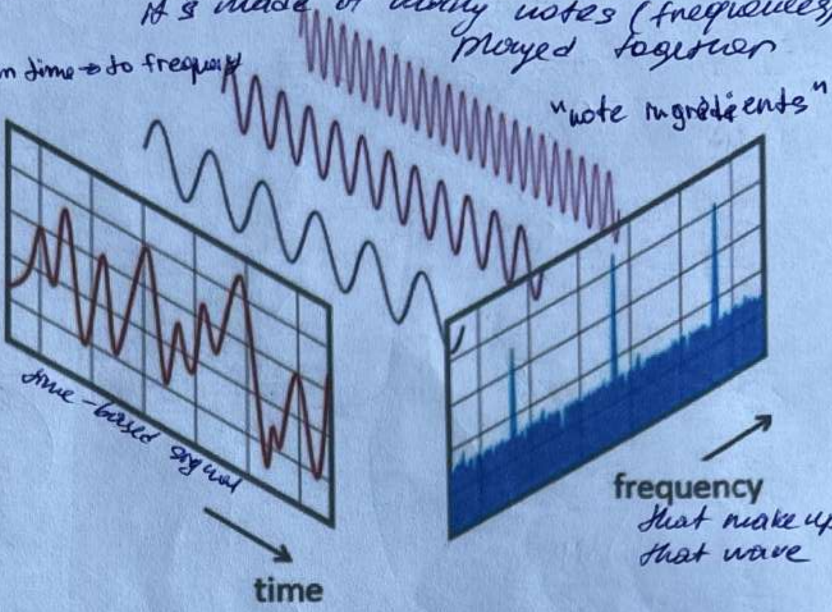
$\boxed{\log_2(n)}$

turns a signal from the time domain to the fr. domain

It's made of many notes (frequencies) played together

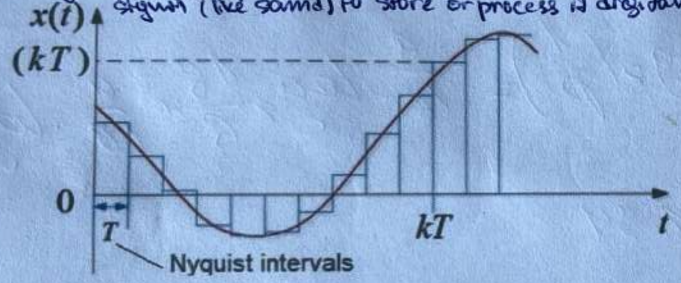
Fourier transform

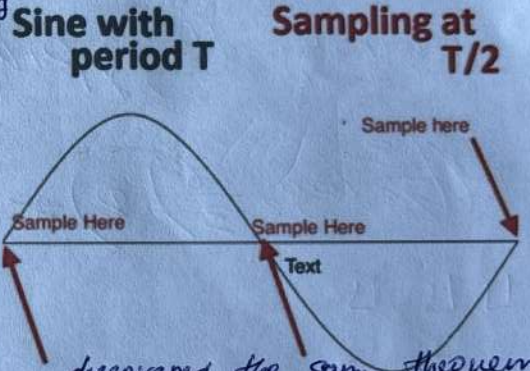
= from time → to frequency



Sampling. Kotelnikov-Nyquist Theorem

taking regular snap shots of a continuous signal (like sound) to store or process it digitally

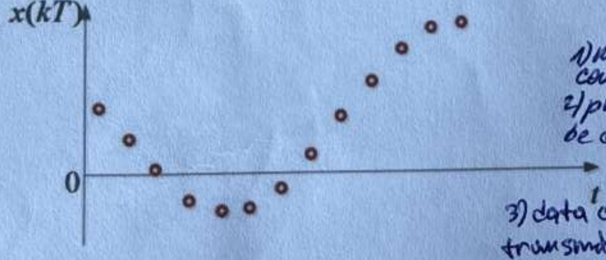




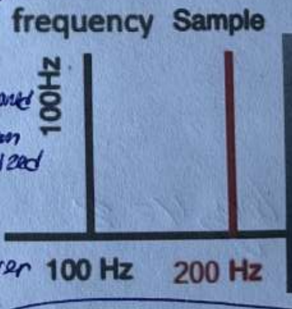
Time intervals T, through which readings s (kT) are taken, are called Nyquist intervals.

discovered the samp. theorem

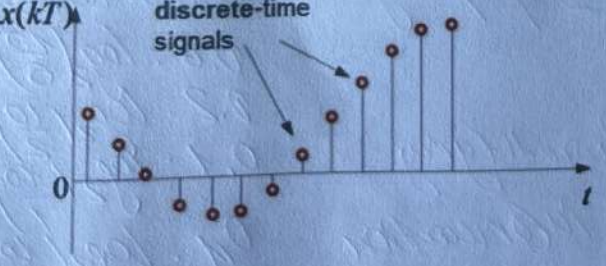
V. Kotelnikov



1) music can be stored
2) phone can be digitalized
3) data can be transmitted over the internet



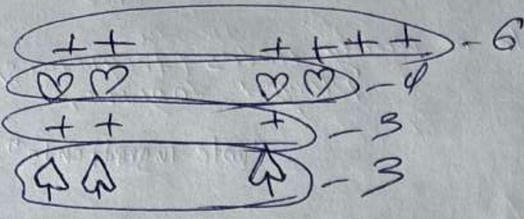
a soviet engineer and scientist



$$F_{sample}>=2*F_{max}$$

$$(T_{sample}<=T_{min}/2)$$

to perfectly capture a signal you must sample at least twice the highest frequency units



$\frac{6}{16} = 1 = \frac{6}{16}$

$\frac{4}{16} = 2 = \frac{3}{16}$

$\frac{3}{16} \cdot 3 = \frac{9}{16}$

$\frac{3}{16} \cdot 3 = \frac{9}{16}$

$\Bigg\} \frac{32}{16} = 2$

$\frac{2}{8} \cdot 1 = \frac{2}{8}$

$\frac{2}{8} \cdot 2 = \frac{4}{8}$

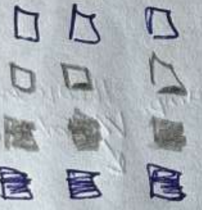
$\frac{2}{8} \cdot 3 = \frac{6}{8}$

$\frac{2}{8} \cdot 3 = \frac{6}{8}$

total: $\frac{18}{8} = \boxed{2,2}$

Entropy:
$0,25 \cdot \log_2 4 +$
$+ 0,25 \cdot \log_2 4 +$
$+ \ldots = \boxed{2}$

Entropy · $H = \sum_{k=1}^{N} p_i \log_2\left(\frac{1}{p_i}\right) = \frac{6}{16} \cdot \log_2 \frac{16}{6} + \frac{4}{16} \cdot \log_2 \frac{16}{4} +$
$+ \left(\left(\frac{3}{16} \log_2 \frac{16}{3}\right) \cdot 2\right) \approx 1,94$



2) Entropy

$\log_2\left(\frac{4}{\frac{1}{4}}\right) = \log_2 4 = 2$

$H = \sum_{x=1}^{4} p(x) \log_2\left(\frac{1}{p(x)}\right)$ – формула Шеннона

$\boxed{I(x) = \log_2\left(\frac{1}{p(x)}\right)}$ Quantifying Information

$H(X) = \sum_{x=1}^{n} p(x) H(x)$

$0.6 \cdot \log_2\left(\frac{10}{6}\right) = 0,4386$
$0.2 \cdot \log_2\left(\frac{10}{2}\right) = 0,4644$
$0.1 \cdot \log_2\left(\frac{10}{1}\right) = 0,3322$
$0.1 \cdot \log_2\left(\frac{10}{1}\right) = 0,3322$

$\sum$ 1,5874

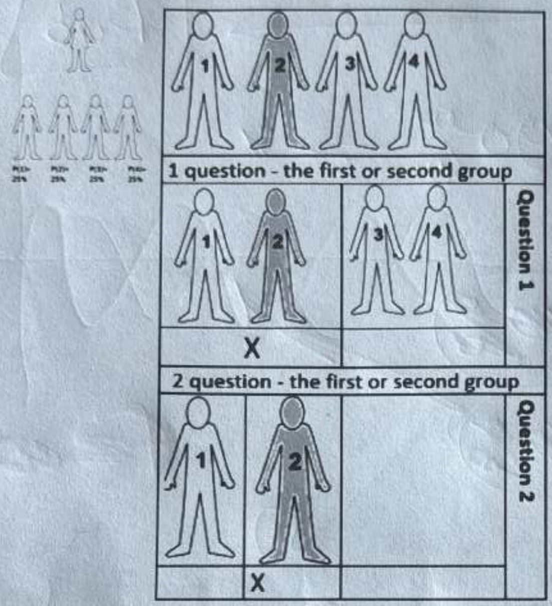
↙ decision strategy for identifying an ind. from a group

Say **NO** to the **first**

Say **YES** to the **second** if it is better than the first

Say **NO** to the **third** only if it is worse than all the others

A binary search like strategy for identification with people grouped based on probability

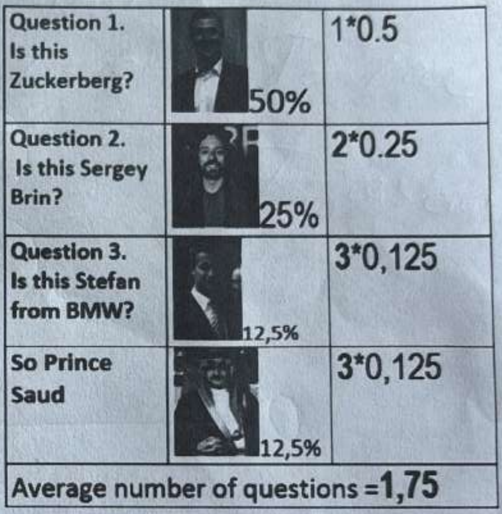


Average number of questions =

2*0.25+ 2*0.25+ 2*0,25+ 2*0,25 =2

Average number of questions =

1*0.5+	2*0.25+	3*0,125+	3*0,125

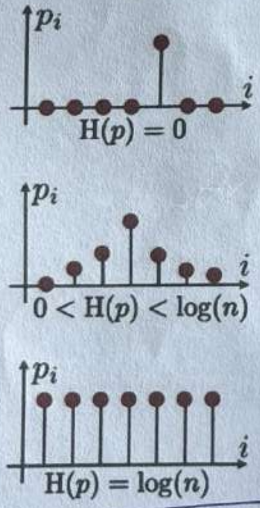


Question 1. Is this Zuckerberg?	50%	1*0.5
Question 2. Is this Sergey Brin?	25%	2*0.25
Question 3. Is this Stefan from BMW?	12,5%	3*0,125
So Prince Saud	12,5%	3*0,125
Average number of questions =**1,75**		

Shannon Entropy

$$H(X) = \sum_{i=1}^{n} p(x_i) \log_b \frac{1}{p(x_i)}$$

$$\sum_{i=1}^{n} p(i) \log_2 \frac{1}{p(i)}$$



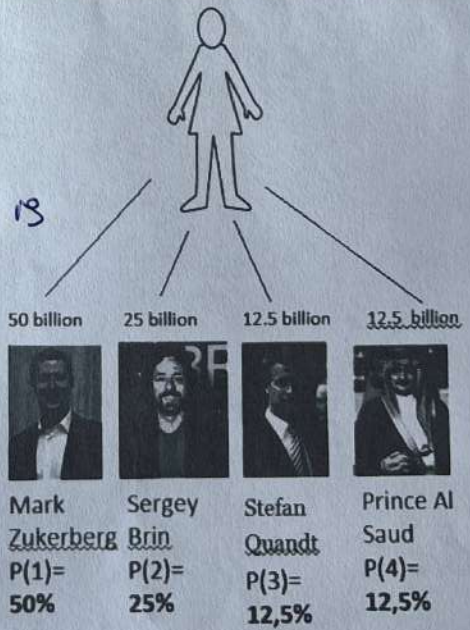
Quantifying information

How surprising the message is

$$I(x_i) = \log_2\left(\frac{1}{p_i}\right)$$

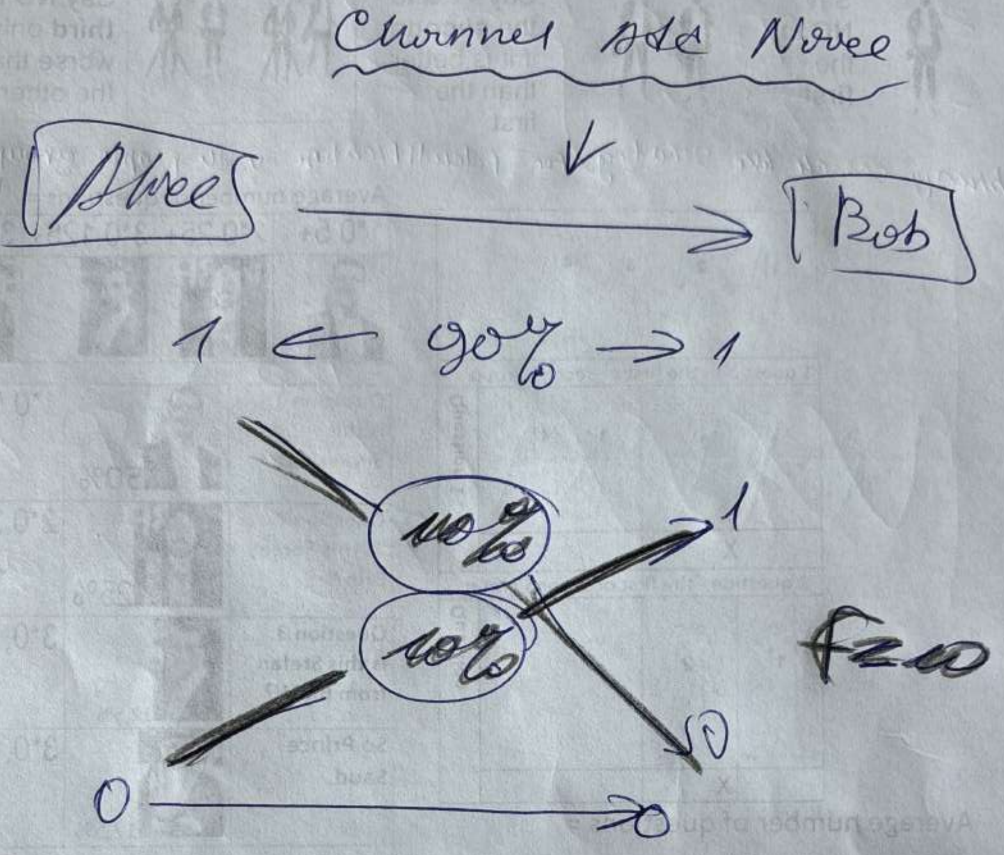
number of bits required to encode choice

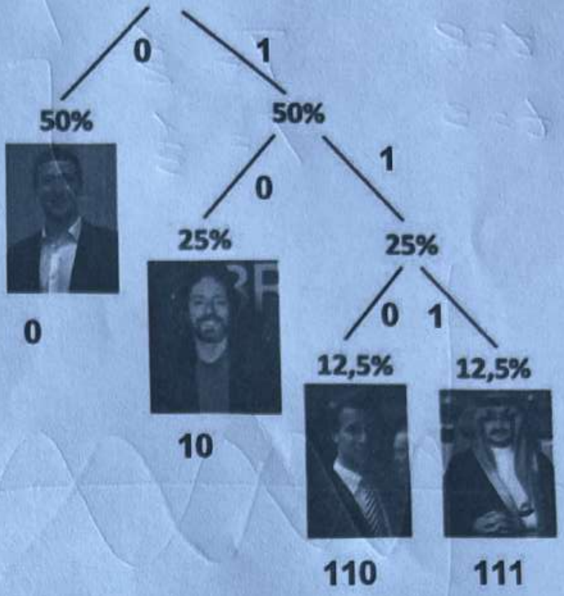
$$\sum_{i=1}^{n} p(x_i)\, I(x_i)$$



This is the maths explanation of Shannon Entropy!

Binary Simmetric



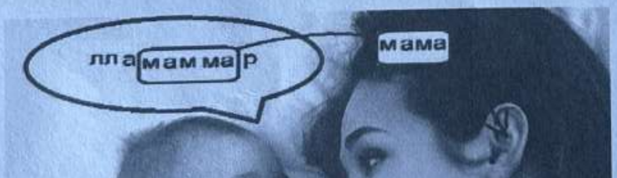


First-order approximation
(symbols independent but with frequencies of Belarusian txt).

Мама мыла ра

М - 3	— 30%	1-3 М
а - 4	— 40%	4-7 а
ы -1	— 10%	8 -ы
л - 1	— 10%	9 -л
р - 1	— 10%	10 -р
10		

лла мамма р



Мама мыла ра

Ма - 2	22%	1-2	ма
ам - 2	22%	3-4	ам
мы - 1	11%	5	мы
ыл - 1	11%	6	ыл
ла - 1	11%	7	ла
а р - 1	11%	8	ар
ра - 1	11%	9	ра
9			

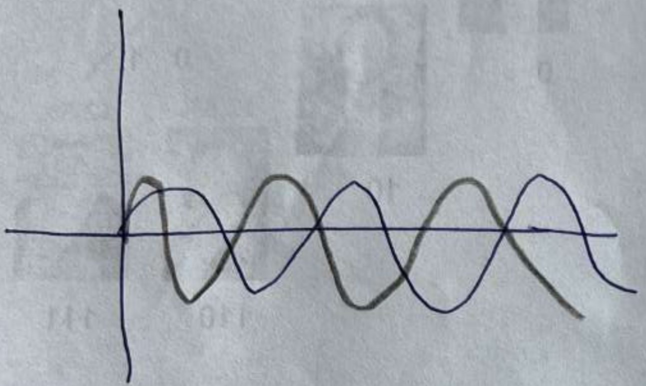
0. 4 6 7 3 1 9 1 6 7 3 5

ам ыл ла ам ма ра ма ыл ла ам мы

мылла рама

Second-order approximation (digram (2-symbols) structure as in Belarusian)

$$\left.\begin{array}{ll} t=1 & T=1 \\ t=2 & T=\frac{1}{2} \\ t=3 & T=\frac{1}{3} \end{array}\right\} T=\frac{1}{t}$$



180

$t=3$ $\quad$ $T=1$ сек

$T=\frac{1}{3}$ $\quad$ $T=0{,}5$ сек